Handling Linguistic Assessments In Life Cycle Costing -
A Fuzzy Approach

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ABSTRACT
This paper outlines a fuzzy set theory approach to handle judgmental linguistic assessments of input parameters in life cycle costing (LCC). This approach is motivated by the fact that in many situations there is a lack of reliable information and data. The proposed methodology is built around an explicit mathematical model. This model is carefully formulated to allow handling of vagueness in all input variables in an efficient and systematic manner. Then, the model is implemented in the form of a computational algorithm. The algorithm handles a number of competing alternatives with ill-defined input data, ranks them automatically, and provides the decision-maker with confidence measures in this ranking. Moreover, it has an efficient built-in procedure that identifies input parameters that have the prominent impact on the quality of the decision. Last but not least, it has the apparent advantage of being transparent which allows more understanding of the decision-making process. The approach is illustrated with an example problem. Finally, directions for further future research are introduced.

Keywords: Decision-Making, Fuzzy Set Theory, Life Cycle Cost (ing), Risk Assessment.

INTRODUCTION
In undertaking a life cycle costing analysis, the analyst employs the discounted cash flow concept (DCF) to calculate the net present value (NPV) of all costs that can emerge during the life cycle of an alternative and the salvage value of that alternative at the end of the analysis period. The analysis is best conducted during the early stages of design where most, if not all, options are open to consideration (Griffin, 1993). However, estimating the input parameters may be difficult at early stages of the design due to lack of data or insight (Mason and Kahn, 1997). In addition, historic data for construction are too small (Edwards and Bowen, 1998) and more importantly are unreliable (Bull, 1993). To overcome data difficulties, a risk assessment technique is usually used. In doing so, either the sensitivity analysis (SA) or a probabilistic technique, usually the Monte Carlo simulation (MCS), is employed. The SA is only effective when the uncertainty in one state variable is predominant. On the other hand, it is required to determine a probability distribution function (PDF) for every uncertain variable to carry out a MCS. Such functions are best derived from statistical analysis of significant data. But, as mentioned previously, historic data are sparse; therefore it is questionable whether statistically meaningful PDFs can be derived.

In the absence of historic data, subjective assessments for the likely values of uncertain input variables have to be elicited from experts. Even if historic data are available, it is common to adjust historic-based assessments with subjective opinions (Sobanjo, 1999). This is because historic data will never provide a good solution and high quality judgement will always be required (Ashworth, 1996). Some researchers claim that it is possible to produce meaningful PDFs using subjective opinions (e.g. Byrne, 1996). However, the authenticity of such assessments is still suspected as Byrne (1997) pointed out.

Bellman and Zadeh (1970) proposed to incorporate the fuzzy set theory (FST) in human decision making. This is because in most decision situations the goals, constraints, and consequences of the proposed alternatives are not known with precision. They argued that very high levels of precision are often neither obtainable nor, more importantly, necessary for an effective analysis of the system. There are two other benefits of employing the FST. First, it is easier to define fuzzy variables than random variables when no or few statistical data are available (Kaufmann and Gupta, 1985; Ferrari and Savoia, 1998). Moreover, mathematical operations on fuzzy sets are much simpler than those required in the framework of probability theory (Ferrari and Savoia, 1998).
Sobanjo (1999) introduced a methodology for handling the subjective uncertainty in life cycle costing (LCC). The model has the apparent advantage of being simple. However, it has two limitations. First, both the interest rate, rehabilitation times, and the analysis period were assumed to be certain. Moreover, only triangular fuzzy numbers were considered in representing uncertain variables. However, an expert should give his estimates together with a choice of the most appropriate membership function for the variable under consideration. Kishk and Al-Hajj (2000a, 2000b) developed a powerful methodology that overcomes these limitations. In a subsequent paper (Kishk and Al-Hajj, 2000c), they extended the methodology to deal with alternatives with different lives.

The methodology outlined in this paper is a part of an on-going project to develop an integrated decision support system (Kishk and Al-Hajj, 1999). In the next section, basic concepts of FST that are necessary for a clear understanding of this paper are summarised. Then, the LCC model is formulated in a way that can handle linguistic assessments of all state variables. The model is then implemented in the form of an efficient computational algorithm. This is followed by an explanation of the proposed methodology in the context of an example problem. Finally, conclusions and further future research are introduced. For convenience of the reader, principal symbols used in the paper are listed in an appendix.

OVERVIEW OF THE FST

Basic Concepts and Definitions

Zadeh (1965) introduced the fuzzy set theory (FST) as an extension of the classical set theory. A fuzzy set, $\tilde{A}$, is one in which the membership has grades in the real continuous interval $[0, 1]$, i.e.

$$\mu_{\tilde{A}}(x) \in [0, 1]$$

As shown in Fig. (1), the end points of the interval $[0, 1]$ conform to no membership ($\mu_{\tilde{A}}(x) = 0$) and full membership ($\mu_{\tilde{A}}(x) = 1$), respectively. However, the infinite number of points in-between these end points can represent various degrees of membership. The closer $\mu_{\tilde{A}}(x)$ is to 1, the more $x$ belongs to $\tilde{A}$; and the closer it is to 0, the less it belongs to $\tilde{A}$. A number of commonly used features of membership functions are depicted in Fig. (1).

![Fig. (1): Features of a fuzzy set.](image)

The $\alpha$-cut of a fuzzy set is a crisp set, $[\alpha a_1, \alpha a_j]$; whose elements have at least $\alpha$ membership, i.e.

$$^{\alpha}A = \{ x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha \}$$

This crisp set may be interpreted as an 'interval of confidence' associated to a level of 'presumption' represented by $\alpha$. According to Kaufmann and Gupta (1985), this interpretation corresponds to the natural mechanism of human thinking in the subjective estimation process.

If a fuzzy set has a height of 1, it is said to be normal. For any $x_1 < x_2 < x_3$ (Fig. 1), a convex fuzzy set is one that satisfies the following relation...
A fuzzy interval is a normal convex fuzzy set. If the core of such set is defined by one point only, it is called a fuzzy number. Fuzzy numbers and intervals are used to approximate imprecise assessments.

**Acquisition of Membership Functions**

The assignment of membership functions (MFs) to fuzzy variables from experts’ judgement is one of the fundamental issues in the FST. The literature is rich in this topic (e.g. Kaufmann and Gupta, 1985; Klir and Yuan, 1995; Pedrycz and Gomide, 1998). In these methods, experts are expected to give answers to some questions necessary to construct the MF of the variable under consideration. For example, Kaufmann and Gupta (1985) proposed the answering of the following three questions for the subjective construction of a membership function.

- What is the smallest value given to the uncertain variable?
- What is the highest value?
- If we are authorised to give one and only one value, what value should be given?

Obviously, this procedure would result in the construction of a triangular fuzzy number (TFN). Then, this TFN can be refined to a subjectively convenient fuzzy number. According to Kaufmann and Gupta (1988), an expert usually gives the smallest value, the highest value and the value with maximum level of presumption. Thus, they pointed out that it is more realistic to use TFNs. However, the use of a trapezoidal fuzzy interval (TFI) allows the introduction of an additional degree of freedom. Fig. (2a) shows the MF of a fuzzy number representing an uncertain cost, $\tilde{C}$, constructed from the expert statement: “the cost is certainly above £900, but is not greater than £1200 and is likely to lie around £1000”. Fig. (2b) depicts the MF of an interval representing a range of estimates without any preference assigned to any of them. In this case, a rectangular MF representing only one ‘interval of confidence’ is sufficient. In the limiting case, if the expert assigns only one value to the variable, implying that it is certain, it is represented by a spike. For example, Fig. (2c) depicts the MF representing the statement: “the cost is definitely £1000”. A trapezoidal MF gives the expert more freedom as he/she can use an interval to represent the likely values. For example, the MF shown in Fig. (2d) represents the statement: “the cost lies approximately between £1000 and £1200, is certainly above £900, but is not greater than £1400”.

A fuzzy number can be constructed to model imprecise estimates such as ‘the cost is about £1000’. In this case, a subjectively convenient support is defined around the approximate value. Then, using linear interpolation a TFN may be constructed as shown in (2e). Similarly, the statement “the cost is from about £800 to about £1000” can be represented by a fuzzy interval as shown in Fig. (2f).
The extension principle (Zadeh, 1975) is a general method of extending non-fuzzy mathematical concepts to deal with fuzzy sets. Several algorithms have been proposed to effectively implement the extension principle. Most of these algorithms originate from interval analysis (Moore, 1966). Each convex fuzzy set can be uniquely represented by closed intervals associated with its $\alpha$-cuts. Thus, operations on real numbers can be extended to fuzzy numbers and intervals. Examples of these algorithms include the DSW algorithm (Dong et al., 1985) and the restricted DSW algorithm (Givens and Tahani, 1987). The restricted DSW algorithm is computationally effective but is valid only for positive intervals. A major drawback of algorithms based on interval analysis is the widening of the resulting function value due to multiple occurrences of variables in the functional expression. Dong and Shah (1987) developed an efficient algorithm known as the vertex method that can effectively overcome this problem.

The solution of fuzzy equations (Kaufmann and Gupta, 1985) is another area of the FST that will be employed in the development of the model. These are equations in which coefficients and unknowns are fuzzy numbers and/or intervals. Consider for example the equation

$$\tilde{A} \tilde{X} = \tilde{B}$$

(4a)

where $\tilde{A}$ and $\tilde{B}$ are fuzzy intervals. The unknown set, $\tilde{X}$, may be attained by solving a set of associated $\alpha$-cuts. For each $\alpha$, the following equation is obtained

$$\alpha \tilde{A} \alpha \tilde{X} = \alpha \tilde{B}$$

(4b)

which have the solution

$$\alpha \tilde{X} = \left[ \frac{\alpha b_l}{\alpha a_r}, \frac{\alpha b_r}{\alpha a_l} \right], \quad \frac{\alpha b_l}{\alpha a_r} \leq \frac{\alpha b_r}{\alpha a_l} \leq \frac{\beta b_l}{\beta a_r} \leq \frac{\beta b_r}{\beta a_l}$$

(5a)

The caveat applied to Eq. (5a) implies that the resulting $\alpha$-cut should be a valid interval. To guarantee that $\tilde{X}$ is convex, the following condition should be valid for each $\alpha \leq \beta$ (Klir and Yuan, 1995)

$$\frac{\alpha b_l}{\alpha a_l} \leq \frac{\alpha b_r}{\alpha a_r} \leq \frac{\beta b_l}{\beta a_l} \leq \frac{\beta b_r}{\beta a_r}$$

(5b)

**Measures Of Uncertainty**

The fuzzy set theory involves two types of uncertainty: non-specificity, $U$, and fuzziness, $F$, (Klir and Yuan, 1995). For normal fuzzy sets, these are calculated as

$$U(\tilde{A}) = \int_0^1 \log \left[ \frac{\alpha a_r - \alpha a_l}{0 a_r - 0 a_l} \right] d\alpha$$

(6)

$$F(\tilde{A}) = \int_{0 a_l}^{0 a_r} 2 \mu_\tilde{A}(x) - 1 dx$$

(7)

where $0 a_l$ and $0 a_r$ are boundaries of the zero-cut (support) of the fuzzy set. Fuzziness and non-specificity are two different types of uncertainty. Moreover, they are totally independent from each other. When non-specificity is reduced, the reduction is viewed as a gain in information, regardless of the associated change in fuzziness. The opposite, however, is not always true (Klir and Yuan, 1995).

**DESIGN OF THE MODEL**

The design of the model involves two principal steps. First, the model is formulated in a formal mathematical framework. Then, it is implemented in the form of a computational algorithm using a fuzzy approach.

**Formulation of the Model**

The model developed by Kishk and Al-Hajj (2000a) calculates the life cycle cost of an alternative as the net present value, $NPV_i$, of all costs and the salvage value of that alternative as
\[ N\bar{PV}_i = \tilde{C}_{0i} + P\bar{WA} \sum_{j=1}^{n_{A}} \tilde{A}_{ij} + \sum_{k=1}^{n_{R}} \tilde{C}_{ik} P\bar{WN}_{ik} - P\bar{WS} \cdot \tilde{S}_i \]  \quad (8)

where
\[ P\bar{WA} = \frac{1}{\bar{T}} \left( 1 - (1 + \bar{T})^{-\bar{v}} \right) \]  \quad (9a)
\[ P\bar{WS} = (1 + \bar{T})^{-\bar{v}} \]  \quad (9b)
\[ P\bar{WN}_{ik} = \frac{1 - (1 + \bar{T})^{-\bar{n}_{ik}\tilde{T}_i}}{(1 + \bar{T})^{\bar{f}_{ik}} - 1} \]  \quad (9c)
\[ \tilde{n}_{ik} = \begin{cases} \text{int}(\frac{\bar{T}}{\bar{f}_{ik}}), & \text{provided that } \text{rem}\left(\frac{\bar{T}}{\bar{f}_{ik}}\right) \neq 0 \\ \frac{\bar{T}}{\bar{f}_{ik}} - 1, & \text{elsewhere} \end{cases} \]  \quad (9d)

Equation (8) may be rewritten as
\[ N\bar{PV}_i = \tilde{C}_{0i} \left( 1 + P\bar{WA} \sum_{j=1}^{n_{A}} \tilde{A}_{ij} + \sum_{k=1}^{n_{R}} \tilde{C}_{ik} P\bar{WN}_{ik} - P\bar{WS} \cdot \tilde{S}_i \right) \]  \quad (10)

where \( \tilde{S}_i, \tilde{A}_{ij}, \) and \( \tilde{C}_{ik} \) are normalised variables given by
\[ \tilde{C}_{0i}\tilde{A}_{ij} = \tilde{A}_{ij} \]  \quad (11a)
\[ \tilde{C}_{0i}\tilde{C}_{ik} = \tilde{C}_{ik} \]  \quad (11b)
\[ \tilde{C}_{0i}\tilde{S}_i = \tilde{S}_i \]  \quad (11c)

In a similar fashion, a normalised net present value, \( N\bar{PV}_i \), may be defined as
\[ \tilde{C}_{0i} N\bar{PV}_i = N\bar{PV}_i \]  \quad (12)

Comparing Eqs. (10) and (12) yields
\[ N\bar{PV}_i = 1 + P\bar{WA} \sum_{j=1}^{n_{A}} \tilde{A}_{ij} + \sum_{k=1}^{n_{R}} \tilde{C}_{ik} P\bar{WN}_{ik} - P\bar{WS} \cdot \tilde{S}_i \]  \quad (13)

which can be written in a compact form as
\[ N\bar{PV}_i = 1 + \sum_{j=1}^{n_{A}} d \tilde{A}R_{ij} + \sum_{k=1}^{n_{R}} d \tilde{N}RC_{ik} - d SAV_i \]  \quad (14)

This normalised value may be seen as an amplification factor applied to the initial cost of an alternative to obtain the life cycle cost of that alternative. Despite this appealing interpretation, \( N\bar{PV}_i \) can’t be used directly to rank alternatives. Rather, it should be modified such that the reference cost is the same for all alternatives. One way to do that is to choose the initial cost for alternative 1. Define a normalised initial cost factor, \( \tilde{I}_i \), given by
\[ \tilde{C}_{0i} \tilde{I}_i = \tilde{C}_{0i} \]  \quad (15)

Substituting from Eqs. (12 and 15) in Eq. (10) and simplifying, yields the net present value normalised in relation to the first alternative, \( N\bar{PV}_i \), as
\[ N\bar{PV}_i = \tilde{I}_i + \sum_{j=1}^{n_{A}} d \tilde{A}R_{ij} + \sum_{k=1}^{n_{R}} d \tilde{N}RC_{ik} - d SAV_i \]  \quad (16)
or in a more compact form as

\[ N\tilde{P}V_i = \tilde{I}_i N\tilde{P}V \]

Model Implementation

Computational Considerations

To guarantee the robustness and efficiency of the computer implementation of the model, four issues were considered. First, the \( \alpha \)-cut method was employed in the computer representation of all fuzzy sets. This representation method is robust and computationally effective than other methods (Ross, 1995). Moreover, it leads to a more understanding of the decision making process (Watson et al., 1979; Kishk and Al-Hajj, 2000c). The second issue was to identify the most appropriate method for carrying out the extended fuzzy operations of the model. A thorough investigation of equations (9-16) reveals that discount factors (Eq. 9) are best computed by the vertex method, while the restricted DSW method is more appropriate for all other computations.

The third issue was to choose an effective ranking procedure. The method proposed by Kaufmann and Gupta (1988) was employed. The method is based on introducing a function \( R \), which maps fuzzy sets to the real line and to use natural ordering. For a fuzzy set \( \tilde{A} \) (Fig. 3), \( R_{\tilde{A}} \) is given by

\[ R_{\tilde{A}} = \frac{1}{2} \left[ \int^{a_r}_{a_l} a \cdot d\alpha \right] = \frac{1}{2} (A_l + A_r) \]

The function \( R \) has an elegant interpretation as an ordinary representative of a fuzzy set.

The fourth issue was to outline appropriate confidence measures to add to the quality of the decision. The concepts of difference and dominance in fuzzy ranking proposed by Tseng and Klein (1989) seems to be useful in this regard. Two fuzzy sets \( \tilde{A} \) and \( \tilde{B} \) are indifferent to each other in their area of overlap (area \( A_1 \) in Fig. 4). In each non-overlap area, however, either \( \tilde{A} \) dominates \( \tilde{B} \) or \( \tilde{B} \) dominates \( \tilde{A} \). The normalised net present values, \( \tilde{N}\tilde{P}V \), of two alternatives are shown in Fig. (4). In areas \( A_2 \), alternative A is better than B, whereas B is better than A in areas \( A_3 \). Thus, the following measures may be defined
The factors \( CI_1 \) and \( CI_2 \) may be interpreted as measures of the confidence in the two statements: ‘A is better than B’ and ‘A is at least as better as B’, respectively.

**The Solution Algorithm**

Based on the issues discussed in previous sections, the following computational algorithm may be proposed (Fig. 5).

1. Experts express their assessments of uncertain state variables as fuzzy numbers and/or intervals, as appropriate. These assessments are drawn with solid lines in Fig. (5).
2. Arrange alternatives such that the first alternative \((i=1)\) is the one with minimum uncertainty in its initial cost, \( \tilde{C}_{i0} \). This cost will be used as a reference point to calculate all normalised net present values, \( \tilde{NPV}_i \).
3. Select an \( \alpha \) such that \( 0 \leq \alpha \leq 1 \).
4. Find the interval in the discount rate, \( \tilde{T} \), that corresponds to this \( \alpha \).
5. Find the corresponding \( \alpha \)-cut in the analysis period, \( \tilde{T} \).
6. Use the vertex method to compute the corresponding intervals in the factors \( P\tilde{WA} \) and \( P\tilde{WS} \) (Eqs. 9a and 9b).
7. Find the intervals in each input membership function representing annual-recurring costs, \( \alpha A_{ij} \), and compute the corresponding interval in the normalised annual cost, \( \alpha \tilde{A}_{ij} \), from Eq. (11a).
8. Use the restricted DSW algorithm to multiply this normalised interval, \( \alpha \tilde{A}_{ij} \), by the corresponding \( \alpha \)-cut in the \( P\tilde{WA} \) factor. This will result in the corresponding discounted \( \alpha \)-cut, \( d\tilde{A}_i R C_{ij} \).
9. Repeat steps 7 and 8 for all annual recurring costs for alternative \( i \) (\( nar_i \) times).
10. Find the intervals in the input membership function representing non-annual recurring cost, $\tilde{C}_{ik}$, and compute the corresponding interval in the normalised non-annual cost, $\tilde{\alpha C}_{ik}$, from Eq. (11b).
11. Find the interval in the frequencies, $\tilde{f}_{ik}$, corresponding to the chosen $\alpha$. Then, use the restricted DSW algorithm to compute the interval in the membership function $\tilde{n}_{ik}$, for the selected $\alpha$-cut level from Eq. (9d).
12. Use the vertex method to compute the interval in the $P\tilde{WN}_{ik}$ factor for the selected $\alpha$-cut level using Eq. (9c).
13. Use the restricted DSW algorithm to multiply the normalised interval by the corresponding $\alpha$-cut in the $P\tilde{WN}_{ik}$ factor. This will result in the corresponding discounted $\alpha$-cut, $dN\tilde{AC}_{ik}$.
14. Repeat steps 10 to 13 for all non-recurring costs for alternative $i$ (nnr times).
15. Find the interval in the input membership function for the salvage value, $\tilde{S}_i$, and compute the corresponding interval in the normalised salvage value, $\tilde{\alpha S}_i$, from Eq. (11c).
16. Use the restricted DSW algorithm to multiply the normalised interval by the corresponding $\alpha$-cut in the $P\tilde{WS}$ factor. This will result in the corresponding discounted $\alpha$-cut, $dS\tilde{AV}_i$.
17. Use Eq. (14) to find the corresponding interval in the $N\tilde{PV}_i$ function.
18. Find the corresponding interval in the normalised initial cost factor, $\tilde{I}_i$, from Eq. (15). For the first alternative, i.e. $i=1$, this interval is simply [1, 1].
19. Using the restricted DSW algorithm, compute the interval for the output membership function, $N\tilde{PV}_i$, for the selected $\alpha$-cut level using Eq. (16).
20. Repeat steps 3-19 for different values of $\alpha$ to complete a smooth $\alpha$-cut representation of $N\tilde{PV}_i$.
21. Repeat steps 3 to 20 for all alternatives.
22. Alternatives are ranked according to their net present values as represented by the $R$ function (Eq. 18).
23. Calculate the measures of confidence using Eqs. (19 and 20).
24. If the decision is unclear regarding two or more alternatives (as defined by a minimum ‘confidence index’ specified by the DM), uncertainty measures (Eqs. 6 and 7) are calculated to identify items that have the dominant contribution to the ambiguity of ranking. The quality of decision can be improved by seeking more ‘precise and specific’ information regarding these items.

**AN EXAMPLE PROBLEM**

In this section, the proposed methodology is illustrated with an example problem. Figure (6) shows the membership functions of the input variables for three design alternatives, respectively. These alternatives are to be ordered according to their life cycle costs for an analysis period, $T$, of 30 years. Using 40 $\alpha$-cuts, the proposed algorithm was employed to rank these alternatives. Figure (7) shows the normalised net present value of the competing alternatives. Despite the relatively small number of employed $\alpha$-cuts, smooth output MFs were obtained. This shows the robustness and computational efficiency of the algorithm. Values of the ranking function, $R$, are 1.70, 2.08, and 1.64 for the original design, alternative A and alternative B, respectively. The measures of confidence, $CI_1$ and $CI_2$, were also calculated and are summarised in Table (1).

<table>
<thead>
<tr>
<th>Rank</th>
<th>Alternatives</th>
<th>Alternative B</th>
<th>Original design</th>
<th>Alternative A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Alternative B</td>
<td>---</td>
<td>0.143</td>
<td>0.527</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.455</td>
<td>0.545</td>
<td>0.7635</td>
</tr>
<tr>
<td>2</td>
<td>Original design</td>
<td>0.053</td>
<td>---</td>
<td>0.399</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.455</td>
<td>---</td>
<td>0.700</td>
</tr>
<tr>
<td>3</td>
<td>Alternative A</td>
<td>0.000</td>
<td>0.236</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.300</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Table (1): Measures of confidence.
Fig. (6): Membership functions for the example problem.
A thorough investigation of these results reveals that both the original design and alternative B have a clear advantage over alternative A. Thus, alternative A can be excluded. However, alternative B was found to be ‘better’ than the original design with a confidence of 0.143; and is ‘at least as better as it’ with a confidence of 0.545. The decision-maker may find these results not conclusive by requiring, for example, a minimum $CI_1$ of 0.6. A more informed decision could be achieved by seeking ‘more precise’ information regarding all input parameters for these two alternatives. However, this may be expensive especially if there are many input variables.

![Graph](https://via.placeholder.com/150)

**Fig. (7): Normalised net present values of**

Alternatively, equations (6 and 7) can be used to calculate uncertainty measures for contributions of various costs and salvage value to the normalised net present value of each alternative. Then, highly uncertain parameters for each alternative can be identified. Table (2) summarises these measures for the original design and alternative B.

<table>
<thead>
<tr>
<th>Costs and Values (Discounted &amp; Normalised)</th>
<th>Alternative B</th>
<th>Original design</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U$</td>
<td>$F$</td>
</tr>
<tr>
<td>Initial cost</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Annual costs</td>
<td>0.200</td>
<td>0.258</td>
</tr>
<tr>
<td>Non-annual repair cost</td>
<td>0.219</td>
<td>0.166</td>
</tr>
<tr>
<td>Salvage value</td>
<td>0.084</td>
<td>0.111</td>
</tr>
</tbody>
</table>

These measures reveal that the non-annual repair cost is the most uncertain parameter for the original design. For alternative B, however, both annual costs and non-annual repair costs should be considered. Figure (8) depicts the revised membership functions for these items. Figure (9) shows the normalised net present value of both alternatives. Values of the ranking function, $R$, are 1.77 and 1.63 for the original design and alternative B, respectively. The measures of confidence, $CI_1$ and $CI_2$, were also calculated and are summarised in Table (3).
Table (3): Revised measures of confidence.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Alternatives</th>
<th>Alternative B</th>
<th>Original design</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$C_{I_1}$</td>
<td>$C_{I_2}$</td>
</tr>
<tr>
<td>1</td>
<td>Alternative B</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>2</td>
<td>Original design</td>
<td>0.004</td>
<td>0.387</td>
</tr>
</tbody>
</table>

Fig. (8): Revised membership functions for the example problem.

Fig. (9): Revised normalised net present values.
These results reveal that alternative B has a clear advantage over the original design and the results of the analysis are conclusive. In other cases, however, the analysis results may not be conclusive, i.e. the minimum confidence level cannot be achieved. In such cases, a more sophisticated analysis including non-monetary benefits would be more appropriate. This will be discussed in more detail in a future paper.

CONCLUSIONS AND FUTURE DIRECTIONS

A novel algorithm was designed around an explicit mathematical LCC model. The algorithm handles subjective assessments of all state variables through a fuzzy approach. It also allows a flexible description of data, analyses it systematically and provides the decision-maker with a better impression of the validity and the usability of these uncertain information and data. These unique features of the algorithm provide more understanding and transparency to the decision making process. This can put the decision-maker in a better position to make an informed decision.

The proposed methodology can be very valuable as a screening device to identify a small number of competing alternatives for further detailed investigation, especially at early stages of the design process. In addition, the specification of the significance of various costs regarding the ambiguity of the decision allows the quick identification of areas that need more detailed information. By focusing on a smaller number of items, the cost of undertaking an LCC analysis can be greatly reduced. Furthermore, it can be employed as an automatic procedure to identify items that have the major impact on the life cycle cost (cost significant items).

It could be argued that some uncertain parameters are better represented by probability distributions. This seems reasonable in cases where meaningful statistics can be derived from historic data for these variables. A considerable research effort is being conducted by the authors to establish appropriate algorithms for combining different representations of uncertainty within the same model calculation. Further work will also focus on dealing with non-monetary benefits. The ultimate objective is to integrate these algorithms in a user-friendly LCC-based decision support system.

REFERENCES


APPENDIX: LIST OF SYMBOLS

\( \tilde{A} \)  
A symbol marked with a tilde represents a fuzzy quantity.

\( \bar{A} \)  
A symbol marked with a bar represents a quantity that is normalised in relation to its initial cost.

\( \hat{A} \)  
A symbol marked with a hat represents a quantity that is normalised in relation to the initial cost of the first alternative.

\( A_1, A_2, A_3 \)  
Areas used to calculate the measures of confidence.

\( A_i, A_j \)  
Areas used to calculate the ranking function, \( R \).

\( \tilde{A}_{ij} \)  
Annual recurring costs of alternative \( i \).

\( ^{\alpha}A \)  
\( \alpha \)-cuts of the fuzzy set \( \tilde{A} \), \( 0 \leq \alpha \leq 1 \). Each \( \alpha \)-cut is a crisp set (an interval).

\( d\tilde{A}RC_i \)  
Discounted annual recurring costs of alternative \( i \).

\( d\tilde{A}RC_i \)  
Discounted annual recurring costs of alternative \( i \), normalised in relation to initial cost of that alternative.

\( d\tilde{A}RC_i \)  
Discounted annual recurring costs of alternative \( i \), normalised in relation to initial cost of alternative 1.

\( a_l, a_u \)  
Left and right branches of the membership function of a fuzzy set, \( \tilde{A} \).

\( [a_l, a_u] \)  
A closed interval where \( a_l \leq a \leq a_u \) representing an \( \alpha \)-cut of a fuzzy set.

\( 0 \)  
The zero-cut (support) of a fuzzy set.

\( C_{0i} \)  
Initial cost of alternative \( i \).
\( \tilde{C}_{ik} \)  
Non-annual recurring costs of alternative \( i \).

\( CI_1, CI_2 \)  
Measures of confidence where \( 0 \leq CI_1 \leq 1 \) and \( 0.5 \leq CI_2 \leq 1 \).

\( F(\tilde{A}) \)  
Measure of fuzziness of a fuzzy set \( \tilde{A} \).

\( \tilde{f}_{ik} \)  
Frequencies of non-annual recurring costs, \( C_{ik} \), of alternative \( i \).

\( \tilde{I}_i \)  
Normalised initial costs in relation to the initial cost of first alternative.

\( \text{int}(a) \)  
Rounds \( a \) to the nearest integer (towards zero).

\( \log(x) \)  
The natural logarithm of \( x \).

\( n_{ik} \)  
Number of recurrences of non-annual recurring costs, \( C_{ik} \), of alternative \( i \).

\( nar \)  
Number of annual recurring costs.

\( nnr \)  
Number of non-annual recurring costs.

\( NPV_i \)  
Net present value of alternative \( i \).

\( \tilde{NPV}_i \)  
Normalised net present value of alternative \( i \), in relation to the initial cost of that alternative.

\( \tilde{NPV} \)  
Normalised net present values in relation to the initial cost of first alternative.

\( dN\tilde{RC}_i \)  
Discounted non-annual recurring costs of alternative \( i \).

\( d\tilde{NRC}_i \)  
Discounted non-annual recurring costs of alternative \( i \), normalised in relation to initial cost of that alternative.

\( \tilde{\tilde{NRC}}_i \)  
Discounted non-annual recurring costs of alternative \( i \), normalised in relation to initial cost of alternative 1.

\( P\tilde{WA} \)  
Present worth factor of annual recurring costs.

\( P\tilde{WN}_k \)  
Present worth factor of a non-annual recurring cost.

\( P\tilde{WS} \)  
Present worth factor for a single future cost.

\( \tilde{r} \)  
Discount rate.

\( R \)  
Ranking function.

\( \text{rem}\left( \frac{a}{b} \right) \)  
Remainder after division of two numbers \( a \) and \( b \).

\( \tilde{S}_i \)  
The salvage value of alternative \( i \), at the end of the analysis period.

\( d\tilde{SAV}_i \)  
Discounted salvage value of alternative \( i \).

\( \tilde{d}\tilde{SAV}_i \)  
Discounted salvage value of alternative \( i \), normalised in relation to initial cost of that alternative.

\( d\tilde{SAV}_i \)  
Discounted salvage value of alternative \( i \), normalised in relation to initial cost of alternative 1.

\( \tilde{T} \)  
Analysis period.

\( U(\tilde{A}) \)  
Measure of non-specificity of a fuzzy set \( \tilde{A} \).

\( \mu^\text{condition}(x) \)  
Degree of membership for the element \( x \) with respect to the fuzzy subset \( \tilde{A} \).

\( |x| \)  
Absolute value of \( x \).

\( \wedge \)  
Minimum of a set of real numbers.

\( \in \)  
Inclusion.