C4 INTEGRATION

Integration is the reverse process to differentiation (described in Calculus Topics C1 - C3). Topic C4 presents the **basic rules of integration** for polynomial functions.

Before studying integration, It is important that you have thoroughly mastered the technique of differentiation using the rules given in Topic C2.

LESSON PLAN

SUMMARY TABLE OF INTEGRATION RULES

RULE	**FUNCTION**	**INTEGRAL**
Power $(n \neq -1)$	x^n	$\frac{x^{n+1}}{n+1} + C$
Constant (A)	A	$Ax + C$
Constant multiple (a) of a function	$a\,f(x)$	$a\int f(x)\,dx$
Sum of functions	$f_1(x) + f_2(x)$	$\int f_1(x)\,dx + \int f_2(x)\,dx$

where C is the arbitrary constant of integration.

Note: The above table summarises the integration rules covered in Topic C4 only.

C4.1 THE INDEFINITE INTEGRAL

Integration is the inverse process to differentiation. This means that given a function f(x), we wish to find a function F(x) such that $F'(x) = f(x)$.

The function F(x) is called the **Indefinite Integral** of f(x) with respect to x and is written as:

$$F(x) = \int f(x)\,dx$$

The integral sign $\int$ indicates that the process of i**ntegration** is being carried out.

The **dx** indicates that we are integrating **with respect to the variable x.**

For example, $\int 3x^2\,dx$

means the indefinite integral of the function $3x^2$ with respect to x.

When the variable name is not x, the dx must be changed to reflect this. For example, the indefinite integral of a function **f(t)** with respect to the variable **t** is written as:

$$\int f(t)\,dt$$

THE ARBITRARY CONSTANT OF INTEGRATION

Since integration is the inverse process to differentiation and differentiating the function $f(x) = x^3$ with respect to x gives $f'(x) = 3x^2$, we can write:

$$\int 3x^2\,dx = x^3$$

However, if $f(x) = x^3 + 1$ then $f'(x) = 3x^2$
Similarly, if $f(x) = x^3 - 3$ then $f'(x) = 3x^2$
and, in general,
if $f(x) = x^3 + \mathbf{C}$ then $f'(x) = 3x^2$ for any constant **C**

Hence the **Indefinite Integral** of $3x^2$ with respect to x, is defined as:

$$\int 3x^2\,dx = x^3 + \mathbf{C}$$

where the constant **C** is called the **Arbitrary Constant of Integration**.

We can check the integration by differentiating with respect to x:

$$\frac{d}{dx}\left(x^3 + C\right) = 3x^2$$

When determining indefinite integrals, it is important always to **remember to include C**, **the arbitrary constant of integration**. Its omission will result in obtaining the incorrect solution in some applications of integration later on.

C4.2 THE INTEGRAL OF A POWER

The rule for integrating a function of the form $\mathbf{f(x) = x^n}$ is:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

where **C** is the arbitrary constant of integration.

This rule applies for **all powers except when n = –1**. The rule for integrating the special case when $f(x) = x^{-1}$ is given in Topic C7 Further Integration.

It may be helpful to remember the rule for integrating a power as:

> **"Increase the power by 1 and divide by the new power"**
>
> **"Remember to add the constant of integration C"**

This rule can be verified by differentiation:

$$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1} + C\right) = \frac{(n+1)\,x^{n+1-1}}{n+1} = \frac{(n+1)\,x^{n}}{n+1} = x^n$$

The above rule applies when **n** is a positive, negative ($n \neq -1$), decimal or fractional power.

EXAMPLE 4.1 **Integrals of Positive Powers**

Integrate with respect to the variable: (i) x^6 (ii) t^2 (iii) x

Solution:

When integrating a positive power the steps to follow are simply:

> (a) **Increase the power by 1**
> (b) **Divide by the new power**
> (c) **Add** the constant of integration **C**

(i) $\int x^6 \, dx = \frac{x^{6+1}}{6+1} + C = \frac{x^7}{7} + C$

(ii) $\int t^2 \, dt = \frac{t^{2+1}}{2+1} + C = \frac{t^3}{3} + C$

(iii) $\int x \, dx = \int x^1 \, dx = \frac{x^{1+1}}{1+1} + C = \frac{x^2}{2} + C$

EXAMPLE 4.2 **Integrals of Negative Powers (except n = –1)**

Integrate with respect to the variable: (i) x^{-2} (ii) $\frac{1}{x^6}$ (iii) $\frac{1}{u^{10}}$

Solution:

We may first need to re-write the function as a single negative power (as shown in examples (ii) and (iii) below), then **integrate** following the steps in the rule:

(a) **Increase the power by 1**
(b) **Divide by the new power**
(c) **Add** the constant of integration **C**

(i) Simply apply rule:

$$\int x^{-2}\,dx = \frac{x^{-2+1}}{-2+1} + C = \frac{x^{-1}}{-1} + C = -x^{-1} + C \qquad = -\frac{1}{x} + C$$

(ii) First, re-write as a single power of x

$$\int \frac{1}{x^6}\,dx = \int x^{-6}\,dx$$

Now, **integrate** with respect to x

$$= \frac{x^{-6+1}}{-6+1} + C = \frac{x^{-5}}{-5} + C$$

Re-write with a positive power

$$= -\frac{1}{5x^5} + C$$

(iii) First, re-write as a single power of u

$$\int \frac{1}{u^{10}}\,du = \int u^{-10}\,du$$

Now, **integrate** with respect to u

$$= \frac{u^{-10+1}}{-10+1} + C = \frac{u^{-9}}{-9} + C$$

Re-write with a positive power

$$= -\frac{1}{9u^9} + C$$

SAQ C4.1 Integrate the following functions with respect to the variable:

(a) x^3 (b) x^7 (c) x^{10} (d) t^{25}

(e) x^{-7} (f) u^{-4} (g) $\frac{1}{x^3}$ (h) $\frac{1}{w^9}$

EXAMPLE 4.3 **Integrals of Decimal and Fractional Powers**

Integrate with respect to the variable:

(i) $x^{2.5}$ (ii) $t^{-\frac{2}{3}}$ (iii) $\sqrt{r}$ (iv) $\frac{1}{\sqrt{x^3}}$

Solution:

(i) Apply the integration rule for powers:

$$\int x^{2.5}\,dx = \frac{x^{2.5+1}}{2.5+1} + C = \frac{x^{3.5}}{3.5} + C$$

(ii) Similarly:

$$\int t^{-\frac{2}{3}}\,dt = \frac{t^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} + C = \frac{t^{\frac{1}{3}}}{\frac{1}{3}} + C = 3t^{\frac{1}{3}} + C$$

(iii) First, change the square-root sign to a fractional power of r

$$\int \sqrt{r}\,dr = \int r^{\frac{1}{2}}\,dr$$

Now, **integrate** with respect to r

$$= \frac{r^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{r^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2r^{\frac{3}{2}}}{3} + C = \frac{2\sqrt{r^3}}{3} + C$$

Note: Since original function $\sqrt{r}$ is written using a square root sign, we express our final answer in the same form, ie. change the fractional index back to a root sign in last step.

(iv) First, change the square-root sign to a fractional power of x

$$\int \frac{1}{\sqrt{x^3}}\,dx = \int \frac{1}{x^{\frac{3}{2}}}\,dx$$

Re-write as a single power

$$= \int x^{-\frac{3}{2}}\,dx$$

Now, **integrate** with respect to x

$$= \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + C$$

$$= \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + C$$

$$= -2x^{-\frac{1}{2}} + C$$

Re-write using a square root sign

$$= -\frac{2}{x^{\frac{1}{2}}} + C$$

$$= -\frac{2}{\sqrt{x}} + C$$

SAQ C4.2 Integrate the following functions with respect to the variable:

(a) $x^{3.5}$ (b) $x^{-1.9}$ (c) $x^{-0.4}$ (d) $t^{\frac{4}{5}}$

(e) $x^{-\frac{7}{4}}$ (f) $\sqrt[3]{u}$ (g) $\sqrt[4]{x^3}$ (h) $\frac{1}{\sqrt[3]{x}}$

C4.3 THE INTEGRAL OF A CONSTANT

The integral of a constant is:

$$\int \mathbf{A}\ \mathbf{dx} = \mathbf{Ax} + \mathbf{C}$$

where **A** is any constant and **C** is the arbitrary constant of integration.

We can verify this rule by differentiation: $\frac{d}{dx}(Ax + C) = A$

EXAMPLE 4.4

Integrate with respect to x: (i) 6 (ii) 2.5 (iii) –1

Solution: (i) $\int 6\ dx = 6x + C$

(ii) $\int 2.5\ dx = 2.5x + C$

(iii) $\int (-1)\ dx = -x + C$

C4.4 THE INTEGRAL OF A CONSTANT MULTIPLE OF A FUNCTION

The rule for integrating a function of the form $\mathbf{ax^n}$, where **a** is the constant multiplier, is:

$$\int \mathbf{ax^n}\ \mathbf{dx} = \frac{\mathbf{ax^{n+1}}}{\mathbf{n+1}} + \mathbf{C}$$

In general, if **a** is any constant, then $\int \mathbf{a\,f(x)\ dx} = \mathbf{a}\int \mathbf{f(x)dx}$

Effectively, this rule means that a constant multiplier can be taken outside the integral. ie.

$$f(x) = x^n \quad \Rightarrow \quad \int a\,x^n\ dx = a\int x^n\ dx$$

EXAMPLE 4.5

Integrate with respect to the variable:

(i) $4x^3$ (ii) $\frac{6}{x^2}$ (iii) $\frac{\sqrt[3]{u}}{2}$

Solution:

(i)
$$\int 4x^3\,dx = 4\int x^3\,dx = 4\left(\frac{x^4}{4}\right) + C = x^4 + C$$

(ii)
$$\int \frac{6}{x^2}\,dx = 6\int \frac{1}{x^2}\,dx = 6\int x^{-2}\,dx = 6\left(\frac{x^{-1}}{-1}\right) + C = -\frac{6}{x} + C$$

(iii)
$$\int \frac{\sqrt[3]{u}}{2}\,du = \frac{1}{2}\int \sqrt[3]{u}\,du = \frac{1}{2}\int u^{\frac{1}{3}}\,du = \frac{1}{2}\left(\frac{3u^{\frac{4}{3}}}{4}\right) + C = \frac{3u^{\frac{4}{3}}}{8} + C = \frac{3\sqrt[3]{u^4}}{8} + C$$

SAQ C4.3 Integrate the following with respect to the variable:

(a) $4x^2$ (b) 3 (c) $7t^6$

(d) $\frac{2}{3}q^{-3}$ (e) $-\frac{2}{x^2}$ (f) $\frac{4}{5\sqrt{x^3}}$

C4.5 THE INTEGRAL OF A SUM (OR DIFFERENCE) OF FUNCTIONS

A function that is the sum of two functions of x can be integrated by adding together the integrals of the two separate functions. The rule is:

$$\int \left(\mathbf{f_1(x) + f_2(x)}\right) \mathbf{dx} = \int \mathbf{f_1(x)\,dx} + \int \mathbf{f_2(x)\,dx}$$

Similarly, the rule for integrating the **difference** of two functions is:

$$\int \left(\mathbf{f_1(x) - f_2(x)}\right) \mathbf{dx} = \int \mathbf{f_1(x)\,dx} - \int \mathbf{f_2(x)\,dx}$$

The sum rule can be extended to apply to any number of separate functions added or subtracted together. In the following examples, note that although each of the integrals has its own arbitrary constant of integration eg. C_1, C_2, these constants are added together to give a single constant of integration, eg. $C = C_1 + C_2$

EXAMPLE 4.6

Integrate with respect to the variable:

(i) $x^4 + x^6$ (ii) $3t^2 + 5t - 9$ (iii) $5x^2 - \dfrac{7}{x^2}$

Solution:

(i) $$\int (x^4 + x^6)\,dx = \int x^4\,dx + \int x^6\,dx = \frac{x^5}{5} + \frac{x^7}{7} + C$$

(ii) $$\int (3t^2 + 5t - 9)\,dt = \int 3t^2\,dt + \int 5t\,dt - \int 9\,dt = t^3 + \frac{5t^2}{2} - 9t + C$$

(iii) $$\int (5x^2 - \frac{7}{x^2})\,dx = \int (5x^2 - 7x^{-2})\,dx = \frac{5x^3}{3} + 7x^{-1} + C = \frac{5x^3}{3} + \frac{7}{x} + C$$

SAQ C4.4 Integrate the following with respect to the variable:

(a) $x^5 + x^7$ (b) $6t^2 - 8t - 7$ (c) $4u^3 - 9u^2 - 4u + 5$

(d) $3x^2 + \dfrac{1}{x^2} - \dfrac{1}{x^3}$ (e) $3\sqrt{w} + \dfrac{2}{\sqrt{w}}$ (f) $x^{-1.4} + 1.4$

C4.6 FURTHER EXAMPLES OF INTEGRATION

The integration rules we have seen so far are:

RULE	FUNCTION	INTEGRAL
Power ($n \neq -1$)	x^n	$\frac{x^{n+1}}{n+1} + C$
Constant (A)	A	$Ax + C$
Constant multiple (a) of a function	$a\,f(x)$	$a\int f(x)\,dx$
Sum of functions	$f_1(x) + f_2(x)$	$\int f_1(x)\,dx + \int f_2(x)\,dx$

where C is the arbitrary constant of integration.

As in differentiation, it is important to be able to recognise situations in which the given function cannot be integrated **directly as it stands** using one or more of the above rules of integration. In such cases the function must be simplified or rearranged into a suitable form for integration.

EXAMPLE 4.7

Find the indefinite integral of the following:

(i) $(2x^2 - 4x)(x^3 + 2x)$ (ii) $\dfrac{3x^4 + 2x^2 - 5}{4x^2}$

Solution:

(i) First multiply out the brackets, then integrate with respect to x:

$$\int (2x^2 - 4)(x^3 + 2x)\ dx = \int (2x^5 - 4x^3 + 4x^3 - 8x)\ dx$$

$$= \int (2x^5 - 8x)\ dx$$

$$= \frac{2x^6}{6} - \frac{8x^2}{2} + C$$

$$= \frac{x^6}{3} - 4x^2 + C$$

(ii) $$\int \left(\frac{3x^4 + 2x^2 - 5}{4x^2} \right) dx$$

First divide all the terms on the top line by $4x^2$ $$= \int \left(\frac{3x^4}{4x^2} + \frac{2x^2}{4x^2} - \frac{5}{4x^2} \right) dx$$

Write each term as a multiple of a single power of x $$= \int \left(\frac{3}{4}x^2 + \frac{1}{2} - \frac{5}{4}x^{-2} \right) dx$$

Now **integrate** with respect to x $$= \frac{3}{4}\left(\frac{x^3}{3} \right) + \left(\frac{1}{2}x \right) - \frac{5}{4}\left(\frac{x^{-1}}{-1} \right) + C$$

$$= \frac{x^3}{4} + \frac{x}{2} + \frac{5}{4x} + C$$

Note: As in differentiation, to integrate a **product** (two functions multiplied together) it is **not** correct just to integrate the factors separately and then to multiply the results together. There is a special rule, called Integration by Parts, which can sometimes be used to integrate a product of functions. This rule is not covered in these notes.

Similarly, to integrate a **quotient** (one function divided by another), it is **not** correct just to integrate the top and bottom lines separately and then to divide one result by the other.

SAQ C4.5

Find the indefinite integral of the following functions with respect to the variable:

(a) $(t^3 + 1)^2$ (b) $y^2\left(\frac{2}{y} - 3 \right)$ (c) $\frac{s-2}{s^3}$ (d) $\frac{x^2 + 3x - 10}{x+5}$

C4.7 THE DEFINITE INTEGRAL

The **Definite Integral** of a function f(x) between the limits of **x = a** and **x = b** is written as:

$$\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a)$$

where **a** is the **lower limit**, **b** is the **upper limit** and

$\mathbf{F(x)} = \int \mathbf{f(x)\,dx}$ is the indefinite integral

The definite integral is **evaluated** by first determining the indefinite integral of f(x). The resulting function F(x) is written inside square brackets with the limits placed outside the right square bracket.

The value of the definite integral is obtained by evaluating **F(b)**, the value of the indefinite integral at **x = b**, and subtracting **F(a),** the value of the indefinite integral at **x = a.**

Hence, the result of evaluating the definite integral is a **number** = **F(b) – F(a).**

The arbitrary constant of integration C always cancels out when limits are applied, eg.

$$\begin{aligned}\int_1^3 (2x+1)\,dx &= \left[x^2 + x + C\right]_1^3 \\ &= \left(3^2 + 3 + C\right) - \left(1^2 + 1 + C\right) \\ &= 12 + C - 2 - C \\ &= 10\end{aligned}$$

Thus, it is not necessary to show the constant C in the evaluation of the definite integral. The calculation is written simply as:

$$\begin{aligned}\int_1^3 (2x+1)\,dx &= \left[x^2 + x\right]_1^3 \\ &= \left(3^2 + 3\right) - \left(1^2 + 1\right) \\ &= 12 - 2 \\ &= 10\end{aligned}$$

Note: It is important to write the limits of the integral the right way round. Interchanging the limits changes the sign of the integral.

EXAMPLE 4.8

Evaluate the following definite integrals:

(i) $\int_0^3 (x^2 + 4)\, dx$ (ii) $\int_{-1}^3 (2x - 1)^2\, dx$

Solution:

(i) $\int_0^3 (x^2 + 4)\, dx = \left[\frac{x^3}{3} + 4x \right]_0^3$

$= \left(\frac{3^3}{3} + 4(3) \right) - \left(\frac{0^3}{3} + 4(0) \right)$

$= (9 + 12) - 0$

$= 21$

(ii) $\int_{-1}^3 (2x - 1)^2\, dx = \int_{-1}^3 (4x^2 - 4x + 1)\, dx$

$= \left[\frac{4x^3}{3} - 2x^2 + x \right]_{-1}^3$

$= \left(\frac{4(3)^3}{3} - 2(3)^2 + 3 \right) - \left(\frac{4(-1)^3}{3} - 2(-1)^2 + (-1) \right)$

$= (36 - 18 + 3) - \left(-\frac{4}{3} - 2 - 1 \right)$

$= 21 + 4\frac{1}{3}$

$= 25\frac{1}{3}$

SAQ C4.6 Evaluate the following definite integrals:

(a) $\int_1^3 (3x^2 + 2x)\, dx$ (b) $\int_{-1}^3 (x^3 - 2x + 2)\, dx$

(c) $\int_0^3 (4 - p)^2\, dp$ (d) $\int_0^4 (r^2 - \sqrt{r})\, dr$

C4.8 SOLUTIONS TO SELF-ASSESSMENT QUESTIONS

SAQ C4.1 Powers

(a) $\int x^3\,dx = \frac{x^4}{4} + C$

(b) $\int x^7\,dx = \frac{x^8}{8} + C$

(c) $\int x^{10}\,dx = \frac{x^{11}}{11} + C$

(d) $\int t^{25}\,dt = \frac{t^{26}}{26} + C$

(e) $\int x^{-7}\,dx = -\frac{x^{-6}}{6} + C$

(f) $\int u^{-4}\,du = -\frac{u^{-3}}{3} + C$

(g) $\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = -\frac{x^{-2}}{2} + C = -\frac{1}{2x^2} + C$

(h) $\int \frac{1}{w^9}\,dw = \int w^{-9}\,dw = -\frac{w^{-8}}{8} + C = -\frac{1}{8w^8} + C$

SAQ C4.2 Decimal and Fractional Powers

(a) $\int x^{3.5}\,dx = \frac{x^{4.5}}{4.5} + C$

(b) $\int x^{-1.9}\,dx = \frac{x^{-1.9+1}}{-1.9+1} = -\frac{x^{-0.9}}{-0.9} + C$

(c) $\int x^{-0.4}\,dx = \frac{x^{-0.4+1}}{-0.4+1} = \frac{x^{0.6}}{0.6} + C$

(d) $\int t^{\frac{4}{5}}\,dt = \frac{t^{\frac{4}{5}+1}}{\frac{4}{5}+1} + C = \frac{5t^{\frac{9}{5}}}{9} + C$

(e) $\int x^{-\frac{7}{4}}\,dx = \frac{x^{-\frac{7}{4}+1}}{-\frac{7}{4}+1} + C = -\frac{4x^{-\frac{3}{4}}}{3} + C$

(f) $\int \sqrt[3]{u}\,du = \int u^{\frac{1}{3}}\,du = \frac{u^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C = \frac{3u^{\frac{4}{3}}}{4} + C$

(g) $\int \sqrt[4]{x^3}\,dx = \int x^{\frac{3}{4}}\,dx = \frac{x^{\frac{3}{4}+1}}{\frac{3}{4}+1} + C = \frac{4x^{\frac{7}{4}}}{7} + C = \frac{4\sqrt[4]{x^7}}{7} + C$

(h) $\int \frac{1}{\sqrt[3]{x}}\,dx = \int x^{-\frac{1}{3}}\,dx = \frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + C = \frac{3x^{\frac{2}{3}}}{2} + C = \frac{3\sqrt[3]{x^2}}{2} + C$

SAQ C4.3 Constants/Constant Multiples

(a) $\int 4x^2\,dx = \frac{4x^3}{3} + C$

(b) $\int 3\,dx = 3x + C$

(c) $\int 7t^6\,dt = 7\left(\frac{t^7}{7}\right) + C = t^7 + C$

(d) $\int \frac{2}{3}q^{-3}\,dq = \frac{2}{3}\left(\frac{q^{-2}}{-2}\right) + C = -\frac{1}{3}q^{-2} + C$

(e) $\int -\frac{2}{x^2}\,dx = -2\int x^{-2}\,dx = -2\left(\frac{x^{-1}}{-1}\right) + C = \frac{2}{x} + C$

(f) $\int \frac{4}{5\sqrt{x^3}}\,dx = \frac{4}{5}\int \frac{1}{x^{\frac{3}{2}}}\,dx = \frac{4}{5}\int x^{-\frac{3}{2}}\,dx = \frac{4}{5}\left(\frac{2x^{-\frac{1}{2}}}{-1}\right) + C = -\frac{8}{5\sqrt{x}} + C$

SAQ C4.4 Sum (or Difference) of Functions

(a) $\int (x^5 + x^7)\,dx = \frac{x^6}{6} + \frac{x^8}{8} + C$

(b) $\int (6t^2 - 8t - 7)\,dt = \frac{6t^3}{3} - \frac{8t^2}{2} - 7t + C = 2t^3 - 4t^2 - 7t + C$

(c) $\int (4u^3 - 9u^2 - 4u + 5)\,du = \frac{4u^4}{4} - \frac{9u^3}{3} - \frac{4u^2}{2} + 5u + C$

$= u^4 - 3u^3 - 2u^2 + 5u + C$

(d) $\int \left(3x^2 + \frac{1}{x^2} - \frac{1}{x^3}\right)dx = \int \left(3x^2 + x^{-2} - x^{-3}\right)dx$

$= x^3 - \frac{1}{x} + \frac{1}{2x^2} + C$

(e) $\int \left(3\sqrt{w} + \frac{2}{\sqrt{w}}\right)dw = \int \left(3w^{\frac{1}{2}} + 2w^{-\frac{1}{2}}\right)dw$

$= 3\left(\frac{2w^{\frac{3}{2}}}{3}\right) + 2\left(\frac{2w^{\frac{1}{2}}}{1}\right) + C$

$= 2\sqrt{w^3} + 4\sqrt{w} + C$

(f) $\int \left(x^{-1.4} + 1.4\right)dx = \frac{x^{-0.4}}{-0.4} + 1.4x + C = -2.5x^{-0.4} + 1.4x + C$

SAQ C4.5 **Further Examples**

(a) $\int (t^3 + 1)^2\, dt = \int (t^6 + 2t^3 + 1)\, dt = \frac{t^7}{7} + \frac{t^4}{2} + t + C$

(b) $\int y^2\left(\frac{2}{y} - 3\right) dy = \int \left(2y - 3y^2\right) dy = y^2 - y^3 + C$

(c) $\int \left(\frac{s-2}{s^3}\right) ds = \int \left(\frac{1}{s^2} - \frac{2}{s^3}\right) ds = \int \left(s^{-2} - 2s^{-3}\right) ds = -\frac{1}{s} + \frac{1}{s^2} + C$

(d) $\int \left(\frac{x^2 + 3x - 10}{x + 5}\right) dx = \int \frac{(x+5)(x-2)}{(x+5)}\, dx = \int (x-2)\, dx = \frac{x^2}{2} - 2x + C$

SAQ C4.6 **The Definite Integral**

(a) $\int_1^3 (3x^2 + 2x)\, dx = \left[x^3 + x^2\right]_1^3 = (3^3 + 3^2) - (1^3 + 1^2) = 36 - 2 = 34$

(b) $\int_{-1}^3 (x^3 - 2x + 2)\, dx = \left[\frac{x^4}{4} - x^2 + 2x\right]_{-1}^3$

$$= \left(\frac{3^4}{4} - 3^2 + 2(3)\right) - \left(\frac{(-1)^4}{4} - (-1)^2 + 2(-1)\right)$$

$$= \left(\frac{81}{4} - 9 + 6\right) - \left(\frac{1}{4} - 1 - 2\right)$$

$$= 17\tfrac{1}{4} + 2\tfrac{3}{4}$$

$$= 20$$

(c) $\int_0^3 (4-p)^2\, dp = \int_0^3 (16 - 8p + p^2)\, dp$

$$= \left[16p - 4p^2 + \frac{p^3}{3}\right]_0^3$$

$$= \left(16(3) - 4(3)^2 + \frac{3^3}{3}\right) - (0)$$

$$= 48 - 36 + 9$$

$$= 21$$

(d) $\int_0^4 \left(r^2 - \sqrt{r}\right) dr = \int_0^4 \left(r^2 - r^{\frac{1}{2}}\right) dr = \left[\frac{r^3}{3} - \frac{2r^{\frac{3}{2}}}{3}\right]_0^4 = \left(\frac{4^3}{3} - \frac{2(4)^{\frac{3}{2}}}{3}\right) - (0)$

$$= \frac{64}{3} - \frac{16}{3}$$

$$= \frac{48}{3}$$

$$= 16$$